Application of Derivatives

Question1

The function $f(x) = \tan x - x$

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Options:

A.

always increases

B.

always decreases

C.

never increases

D.

neither increases nor decreases

Answer: A

Solution:

The function given is $f(x) = \tan x - x$.

To determine whether this function is increasing, decreasing, or neither, we need to analyze its derivative:

$f'(x) = \sec^2 x - 1 = \tan^2 x$

Since $\tan^2 x$ is always greater than or equal to zero ($\tan^2 x \geq 0$), the derivative $f'(x)$ is non-negative across its domain. This implies that the function $f(x)$ is always increasing or at the very least, non-decreasing.

Thus, the function $f(x) = \tan x - x$ consistently increases.

Question2

The length of a rectangle is five times the breadth. If the minimum perimeter of the rectangle is 180 cm , then

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Options:

A. Breadth $\leq$ 15 cm

B. Breadth $\geq$ 15 cm

C. Length $\leq$ 15 cm

D. Length $=$ 15 cm

Answer: B

Solution:

Since, perimeter ≥ 180
$\Rightarrow 2$ (length $+$ breadth ≥ 180
$\Rightarrow 5$ breadth $+$ breadth ≥ 90
$[\because$ length $= 5$ breadth $]$
$\Rightarrow 6$ breadth ≥ 90
Hence, breadth ≥ 15

Question3

The value of C in $(0, 2)$ satisfying the mean value theorem for the function $f(x) = x(x-1)^2, x \in [0, 2]$ is equal to

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Options:

A. $3/4$

B. 4/3

C. 1/3

D. 2/3

Answer: B

Solution:

$f(x)=x(x-1)^2; x \in [0,2]$

$$f'(c)=\frac{f(b)-f(a)}{b-a}; f(2)=2, f(0)=0$$

$$\begin{aligned}\therefore f'(x) &= (x-1)^2+2x(x-1)\\ &= x^2+1-2x+2x^2-2x\\ &= 3x^2-4x+1\end{aligned}$$

So, $f'(c)=3c^2-4c+1$

Thus, $3c^2-4c+1=\frac{f(2)-(0)}{2-0}=\frac{2-0}{2-0}=1$

$\Rightarrow \quad c(3c-4)=0$

Hence, $c=\frac{4}{3}$

Question4

For the function $f(x)=x^3-6x^2+12x-3; x=2$ is

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Options:

A. A point of minimum

B. A point of inflexion

C. Not a critical point

D. A point of maximum

Answer: B

Solution:

$\because f(x) = x^3 - 6x^2 + 12x - 3$
$\Rightarrow f'(x) = 3x^2 - 12x + 12$
$\Rightarrow f''(x) = 6x - 12 = 6(x - 2)$
$\Rightarrow f''(2) = 0$
Hence, $x = 2$ is a point of inflexion.

Question5

The function $x^x; x > 0$ is strictly increasing at

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Options:

A. $\forall x \in R$

B. $x < \frac{1}{e}$

C. $x > \frac{1}{e}$

D. $x < 0$

Answer: C

Solution:

Let $y = x^x$

$\frac{dy}{dx} = x^x(1 + \log x)$

For increasing function, $\frac{dy}{dx} > 0$

$\Rightarrow \quad x^x(1 + \log x) > 0$
$\Rightarrow \quad 1 + \log x > 0$
$\Rightarrow \quad \log_e x > \log_e \frac{1}{e}$
Hence, $\quad x > \frac{1}{e}$

Question6

The maximum volume of the right circular cone with slant height 6 units is

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Options:

A. $4\sqrt{3}\pi$ cu units

B. $16\sqrt{3}\pi$cu units

C. $3\sqrt{3}\pi$ cu units

D. $6\sqrt{3}\pi$ cu units

Answer: B

Solution:

$\because$ Slant height of the cone, $L = 6$ units.

Let the radius be r and height be h.

$$\begin{aligned} \text{and volume } (V) &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi\left(L^2 - h^2\right)h \quad \left[\because L^2 = r^2 + h^2\right] \\ &= \frac{1}{3}\pi\left(36 - h^2\right)h \end{aligned}$$

So, $\frac{dV}{dh} = \frac{1}{3}\pi\left(36 - 3h^2\right) = 0 \Rightarrow h = 2\sqrt{3}$ units

Now, $\frac{d^2V}{dh^2} = -2\pi h < 0$ at $h = 2\sqrt{3}$ units

Hence, the maximum volume of the cone

$$\begin{aligned} V &= \frac{1}{3}\pi\left(36 - h^2\right)h \\ &= \frac{1}{3}\pi\left(36 - (2\sqrt{3})^2\right)2\sqrt{3} \\ &= 16\sqrt{3}\pi \text{ cu units} \end{aligned}$$

Question7

If $f(x) = xe^{x(1-x)}$, then $f(x)$ is

KCET 2024

Options:

A. increasing in R

B. decreasing in R

C. decreasing in $\left[-\frac{1}{2}, 1\right]$

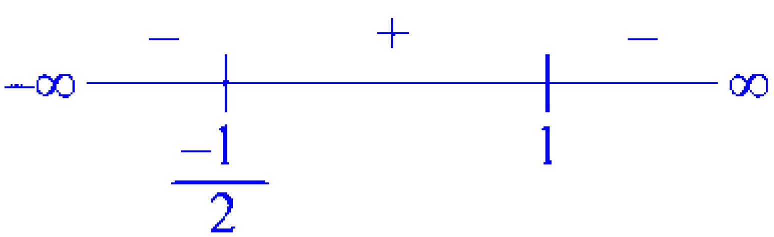
D. increasing in $\left[-\frac{1}{2}, 1\right]$

Answer: D

Solution:

Given,

$$
\begin{aligned}
f(x) &= xe^{x(1-x)} \\
f'(x) &= e^{x(1-x)} + xe^{x(1-x)}(1-2x) \\
&= e^{x(1-x)}(1 + x(1-2x)) \\
&= -e^{x(1-x)}\left(2x^2 - x - 1\right) \\
&= -e^{x(1-x)}\left(2x^2 - 2x + x - 1\right) \\
&= -e^{x(1-x)}(x-1)(2x+1)
\end{aligned}
$$



So, $f(x)$ is increasing when $f'(x) \geq 0$ and decreasing when $f'(x) \leq 0$.

Hence, f is increasing in $\left[\frac{-1}{2}, 1\right]$.

Question8

If $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{du}{dv}$ is

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Options:

A. 2

B. $\frac{1-x^2}{1+x^2}$

C. 1

D. $\frac{1}{2}$

Answer: C

Solution:

$$\text{Here, } u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$u = 2\tan^{-1}x \left[\because 2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]$$

$$\Rightarrow \frac{du}{dx} = 2 \times \frac{1}{1+x^2} \quad \text{.... (i)}$$

$$\text{and } v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$y = 2\tan^{-1}x \quad \left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$\Rightarrow \quad \frac{dv}{dx} = \frac{2}{1+x^2} \quad \text{.... (ii)}$

From Eqs. (i) and (ii), we get

$\frac{dv}{dx} = 1$

Question9

The distance 's' in meters travelled by a particle in 't' seconds is given by $s = \frac{2t^3}{3} - 18t + \frac{5}{3}$. The acceleration when the particle comes to rest is :

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Options:

A. $10 \ \mathrm{m}^2/\mathrm{s}$

B. $12 \ \mathrm{m}^2/\mathrm{s}$

C. $18 \ \mathrm{m}^2/\mathrm{s}$

D. $3 \ \mathrm{m}^2/\mathrm{s}$

Answer: B

Solution:

To find the acceleration when the particle comes to rest, we first have to determine at what time 't' the particle comes to rest. A particle comes to rest when its velocity is zero. The velocity 'v' of a particle is given by the derivative of the position function with respect to time, which in this case is

$v(t) = \frac{ds}{dt}$

Let us find the velocity function by differentiating the given position function:

$v(t) = \frac{d}{dt}\left(\frac{2t^3}{3} - 18t + \frac{5}{3}\right) = \frac{d}{dt}\left(\frac{2t^3}{3}\right) - \frac{d}{dt}(18t) + \frac{d}{dt}\left(\frac{5}{3}\right)$

Applying the differentiation rules, we get:

$v(t) = \frac{2}{3} \cdot 3t^2 - 18 = 2t^2 - 18$

Now, we will set the velocity to zero to find the time 't' when the particle comes to rest:

$0 = v(t) = 2t^2 - 18$

Solving for 't' gives:

$2t^2 = 18$

$t^2 = \frac{18}{2}$

$t^2 = 9$

$t = \pm 3$

Since time cannot be negative, we consider '$t = 3$' seconds.

Now we need to find the acceleration at t=3 seconds. Acceleration 'a' is the derivative of the velocity function, i.e.,

$a(t) = \frac{dv}{dt}$

Let us differentiate the velocity function to find the acceleration function:

$a(t) = \frac{d}{dt}(2t^2 - 18) = \frac{d}{dt}(2t^2) - \frac{d}{dt}(18)$

Applying the differentiation rules, we get:

$a(t) = 2 \cdot 2t - 0 = 4t$

Finally, we find the acceleration when the particle comes to rest, i.e. at $t = 3$ seconds:

$a(3) = 4 \cdot 3 = 12 \text{ m/s}^2$

Therefore, the acceleration when the particle comes to rest is 12 m/s^2, which corresponds to Option B.

Question10

A particle moves along the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$. When the rate of change of abscissa is 4 times that of its ordinate, then the quadrant in which the particle lies is

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Options:

A. II or IV

B. III or IV

C. II or III

D. I or III

Answer: D

Solution:

Given, particles moves along the curve

$\frac{x^2}{16} + \frac{y^2}{4} = 1$

$$\Rightarrow \frac{2x}{16} + \frac{2y}{4}\frac{dy}{dx} = 0$$

$$\therefore \quad \frac{dy}{dx} = \frac{-2x}{16} \times \frac{4}{2y} = \frac{-x}{4y} \quad \text{.... (i)}$$

According to the question, $x = 4y$

From Eq. (i), we get

$\frac{dy}{dx} = -1$

When slope is negative, then the particle lies in Ist and IIIrd quadrant.

Question11

An enemy fighter jet is flying along the curve, given by $y = x^2 + 2$. A soldier is placed at $(3, 2)$ wants to shoot down the jet when it is nearest to him. Then, the nearest distance is

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Options:

A. $\sqrt{6}$ units

B. 2 units

C. $\sqrt{5}$ units

D. $\sqrt{3}$ units

Answer: C

Solution:

Let $P(x, y)$ be the position of the jet and the solider is placed at $A(3, 2)$.

$\Rightarrow \quad AP = \sqrt{(x-3)^2 + (y-2)^2}$ .... (i)

As, $\quad y = \left(x^2 + 2\right) \Rightarrow x^2 = y - 2$ .... (ii)

$$\Rightarrow (AP)^2 = (x-3)^2 + x^4 = z \quad \text{(say)}$$
$$\Rightarrow \frac{dz}{dx} = 2(x-3) + 4x^3 \text{ and } \frac{d^2z}{dx^2} = 2 + 12x^2$$

For local points of maxima and minima

$$\frac{dz}{dx} = 0 \Rightarrow 2(x-3) + 4x^3 = 0$$

$$x = 1 \text{ and } \frac{d^2z}{dx^2}(\text{ at } x = 1) = 14 > 0$$

$\therefore$ z is minimum, where $x = 1, y = 1 + 2 = 3$

Also, minimum distance

$= \sqrt{(1-3)^2 + (3-2)^2} = \sqrt{5}$ units

Question12

A circular plate of radius 5 cm is heated. Due to expansion, its radius increase at the rate of 0.05 cm/s. The rate at which its area is increasing when the radius is 5.2 cm is

KCET 2023

Options:

A. $27.4\,\pi \text{ cm}^2/\text{s}$

B. $5.05\,\pi \text{ cm}^2/\text{s}$

C. $0.52\,\pi \text{ cm}^2/\text{s}$

D. $5.2\,\pi \text{ cm}^2/\text{s}$

Answer: C

Solution:

Let r be the radius of the circular disc and A be the area of the circular disc at any instant of time. we know that

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

According to the question, the circular disc expands on heating with the rate of change of radius is 0.05 cm/s

$$\frac{dr}{dt} = 0.05 \text{ cm/s}$$
$$\frac{dA}{dt} = 2 \times \pi \times 5.2 \times 0.05$$
$$\Rightarrow \frac{dA}{dt} = 0.52\,\pi \text{ cm}^2/\text{s}$$

So, the rate at which its area is increasing, when radius is 5.2 cm is $0.52\,\pi \text{ cm}^2/\text{s}$.

Question13

The function $f(x) = \log(1+x) - \frac{2x}{2+x}$ is increasing on

KCET 2022

Options:

A. $(-\infty, \infty)$

B. $(\infty, -1)$

C. $(-1, \infty)$

D. $(-\infty, 0)$

Answer: C

Solution:

Given, $f(x) = \log(1+x) - \frac{2x}{2+x}$

Differentiating the function w.r.t. x, we get

$$f'(x) = \frac{1}{(1+x)}(0+1) - 2\left[\frac{(2+x)\times 1 - x(0+1)}{(2+x)^2}\right]$$
$$= \frac{1}{1+x} - 2\left[\frac{2+x-x}{(2+x)^2}\right] = \frac{1}{1+x} - \frac{4}{(2+x)^2}$$
$$= \frac{(2+x)^2 - 4(1+x)}{(1+x)(2+x)^2} = \frac{4+x^2+4x-4-4x}{(x+1)(x+2)^2}$$
$$= \frac{x^2}{(x+1)(x+2)^2} = \left(\frac{x}{x+2}\right)^2 \cdot \frac{1}{(x+1)}$$

Sign of $f'(x)$ depends on the sign of $\frac{1}{(x+1)}$

$f'(x) > 0$ when $x > -1 \Rightarrow$ increasing

$f'(x) < 0$ when $x < -1 \Rightarrow$ decreasing

Hence, $f(x)$ is increasing on $(-1, \infty)$.

Question14

The coordinates of the point on the $\sqrt{x} + \sqrt{y} = 6$ at which the tangent is equally inclined to the axes is

KCET 2022

Options:

A. $(4, 4)$

B. $(1, 1)$

C. $(9, 9)$

D. $(6, 6)$

Answer: C

Solution:

Given, $\sqrt{x} + \sqrt{y} = 6$

$\sqrt{x} + \sqrt{y} = 6$

Differentiating w.r.t. x, we get

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$

Since, the tangent is equally inclined to the axes.

$$\frac{dy}{dx} = \pm 1 \Rightarrow -\frac{\sqrt{y}}{\sqrt{x}} = \pm 1$$
$$\Rightarrow \quad \sqrt{y} = \pm\sqrt{x} \Rightarrow y = x$$

Therefore, $\sqrt{x} + \sqrt{y} = 6 \Rightarrow \sqrt{x} + \sqrt{x} = 6 \Rightarrow 2\sqrt{x} = 6$

$\Rightarrow \quad \sqrt{x} = 3 \Rightarrow x = 9$
$\therefore \quad x = 9$ and $y = 9$

Question15

The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly

KCET 2022

Options:

A. decreasing in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

B. decreasing in $\left[0, \frac{\pi}{2}\right]$

C. increasing in $\left(\pi, \frac{3\pi}{2}\right)$

D. decreasing in $\left(\frac{\pi}{2}, \pi\right)$

Answer: D

Solution:

Given, $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$

$$f'(x) = 12\sin^2 x \cdot \cos x - 12\sin x\cos x + 12\cos x$$
$$= 12\cos x\left[\sin^2 x - \sin x + 1\right]$$
$$= 12\cos x\left[\sin^2 x + (1 - \sin x)\right]$$

We know, $1 - \sin x \geq 0$ and $\sin^2 x \geq 0$

$\therefore \sin^2 x + 1 - \sin x > 0$

Therefore, $f'(x) > 0$, when $\cos x > 0 \Rightarrow x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

So, $f(x)$ is increasing when $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $f'(x) < 0$, when $\cos x < 0 \Rightarrow x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

So, $f(x)$ is decreasing when $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Hence, $f(x)$ is decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

Question16

The cost and revenue functions of a product are given by $c(x) = 20x + 4000$ and $R(x) = 60x + 2000$ respectively, where x is the number of items produced and sold. The value of x to earn profit is

KCET 2021

Options:

A. > 50

B. > 60

C. > 80

D. > 40

Answer: A

Solution:

Given, $C(x) = 20x + 4000$

$R(x) = 60x + 2000$

$x =$ number of items produced and sold

To earn profit, $R(x) - C(x) > 0$

$$\Rightarrow \quad 60x + 2000 - 20x - 4000 > 0$$
$$\Rightarrow \quad 40x - 2000 > 0$$
$$\Rightarrow \quad 40x > 2000$$
$$\Rightarrow \quad x > 50$$

Question17

A particle starts form rest and its angular displacement (in radians) is given by $\theta = \frac{t^2}{20} + \frac{t}{5}$. If the angular velocity at the end of $t = 4$ is k, then the value of $5k$ is

KCET 2021

Options:

A. 0.6

B. 5

C. 5k

D. 3

Answer: D

Solution:

$\theta = \frac{t^2}{20} + \frac{t}{5}$

On differentiating θ w.r.t. t,

$$\frac{d\theta}{dt} = \frac{t}{10} + \frac{1}{5}$$
$$\left(\frac{d\theta}{dt}\right)_{t=4} = \frac{4}{10} + \frac{1}{5}$$
$$\Rightarrow \quad k = \frac{3}{5}$$
$$\therefore \quad 5k = 3$$

Question18

The function $f(x) = x^2 - 2x$ is strictly decreasing in the interval

KCET 2021

Options:

A. $(-\infty, 1)$

B. $(1, \infty)$

C. R

D. $(-\infty, \infty)$

Answer: A

Solution:

$f(x) = x^2 - 2x$
$\therefore f'(x) = 2x - 2$

$f(x)$ is strictly decreasing, when $f'(x) < 0$

$$\begin{aligned} & f'(x) < 0 \\ \Rightarrow \quad & 2(x-1) < 0 \\ \Rightarrow \quad & x < 1 \end{aligned}$$

Hence, $f(x)$ is strictly decreasing in the interval $(-\infty, 1)$.

Question19

The maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$ is

KCET 2021

Options:

A. 1

B. 23

C. 5

D. -23

Answer: C

Solution:

$y = -x^3 + 3x^2 + 2x - 27$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = -3x^2 + 6x + 2$

Slope of the curve $= -3x^2 + 6x + 2$

Let $\quad m = -3x^2 + 6x + 2$

$\therefore \quad \frac{dm}{dx} = -6x + 6$

Put $\frac{dm}{dx} = 0$ to find the critical points,

$\Rightarrow \quad -6x + 6 = 0$
$\Rightarrow \quad x = 1$
$\quad \left(\frac{d^2m}{dx^2}\right)_{x=1} = -6 < 0$

Slope is maximum at $x = 1$

The maximum slope,

$$m_{\max} = -3(1)^2 + 6(l) + 2$$
$$= -3 + 6 + 2 = 5$$

Question20

If the curves $2x = y^2$ and $2xy = K$ intersect perpendicularly, then the value of K^2 is

KCET 2020

Options:

A. 4

B. $2\sqrt{2}$

C. 2

D. 8

Answer: D

Solution:

Given curves are

$2x = y^2 \quad$ .... (i)

and $2xy = K \quad$ .... (ii)

On solving both Eqs. (i) and (ii), we get

$x=\frac{K^{2/3}}{2}$ and $=K^{1/3}$

$\therefore$ Intersecting point of both curves is $\left(\frac{K^{2/3}}{2}, K^{1/3}\right)$

Now differentiate Eq. (i) w.r.t. x, we get

$$2=2y\frac{dy}{dx}$$
$$\Rightarrow \quad \frac{dy}{dx}=\frac{1}{y}$$

$\therefore$ Slope of tangent at $\left(\frac{K^{2/3}}{2}, K^{1/3}\right)=\frac{1}{K^{1/3}}$

And differentiate Eq. (ii) w.r.t. x, we get

$2\left(x\frac{dy}{dx}+y\right)=0 \Rightarrow \frac{dy}{dx}=\frac{-y}{x}$

$\therefore$ Slope of tangent at $\left(\frac{K^{2/3}}{2}, K^{1/3}\right)$

$=\frac{-K^{1/3}}{K^{2/3}/2}=-2K^{-1/3}$

Since, both curves intersect perpendicularly

$$\therefore \quad \frac{1}{K^{1/3}} \times\left(-2K^{-1/3}\right)=-1$$
$$\Rightarrow -2K^{-2/3}=-1 \Rightarrow K^{2/3}=2 \Rightarrow K^2=8$$

Question21

If the side of a cube is increased by 5%, then the surface area of a cube is increased by

KCET 2020

Options:

A. 10%

B. 60%

C. 6%

D. 20%

Answer: A

Solution:

We know that the surface area of cube is given by $S = 6x^2$

$$\therefore \quad \frac{dS}{dx} = 12x^2$$
$$\Delta x = 5\% \text{ of } x = 0.05x$$

The surface area of a cube is increased by $= \frac{\Delta S}{S} \times 100\%$

$$= \frac{\left(\frac{dS}{dx}\right)\Delta x}{S} \times 100\%$$
$$= \frac{12x \times (0.05x)}{6x^2} \times 100\%$$
$$= 0.10 \times 100\% = 10\%$$

Question22

The maximum value of $\frac{\log_e x}{x}$, if $x > 0$ is

KCET 2020

Options:

A. e

B. 1

C. $\frac{1}{e}$

D. $-\frac{1}{e}$

Answer: C

Solution:

Let $y = \frac{\log_e x}{x}$

$\Rightarrow \quad \frac{dy}{dx}=\frac{x\cdot\frac{1}{x}-\log_e x\cdot 1}{x^2}=\frac{1-\log_e x}{x^2}$

For maximum, put $\frac{dy}{dx}=0$

$$\begin{aligned}
\Rightarrow \quad 1-\log_e x &= 0 \\
\Rightarrow \quad \log_e x &= 1 \\
\Rightarrow \quad x &= e
\end{aligned}$$

$$\text{Now, } \frac{d^2y}{dx^2}=\frac{x^2\left(0-\frac{1}{x}\right)-(1-\log_e x)(2x)}{x^4}$$

$$=\frac{-x-2x\left(1-\log_e x\right)}{x^4}$$

$$\therefore \quad \left(\frac{d^2y}{dx^2}\right)_{\text{at } x=e}<0$$

$$\therefore \quad y(e)=\frac{\log_e e}{e}=\frac{1}{e}$$

Question23

The interval in which the function $f(x)=x^3-6x^2+9x+10$ is increasing in

KCET 2019

Options:

A. $[1,3]$

B. $(-\infty,1)\cup(3,\infty)$

C. $(-\infty,-1]\cup[3,\infty)$

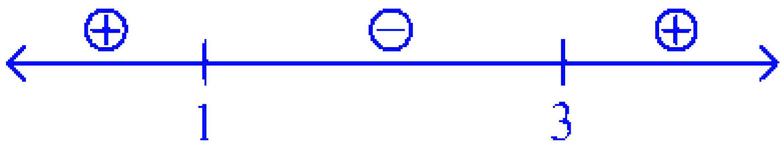
D. $(-\infty,1]\cup[3,\infty)$

Answer: D

Solution:

We have, $f(x)=x^3-6x^2+9x+10$

$$\begin{aligned}
\Rightarrow f'(x) &= 3x^2-12x+9=3\left(x^2-4x+3\right) \\
&= 3(x-1)(x-3)
\end{aligned}$$



Here, $f'(x) \geq 0$ if $x \in (-\infty, 1) \cup (3, \infty)$

Question24

The sides of an equilateral triangle are increasing at the rate of $4\ \mathrm{cm/sec}$. The rate at which its area is increasing, when the side is $14\ \mathrm{cm}$

KCET 2019

Options:

A. $42\ \mathrm{cm}^2/\mathrm{sec}$

B. $10\sqrt{3}\ \mathrm{cm}^2/\mathrm{sec}$

C. $14\ \mathrm{cm}^2/\mathrm{sec}$

D. $14\sqrt{3}\ \mathrm{cm}^2/\mathrm{sec}$

Answer: A

Solution:

We have, $\frac{dx}{dt} = 4\ \mathrm{cm/sec}$

Area of equilateral triangle, $A = \frac{\sqrt{3}}{4}x^2$

$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{4}(2x)\frac{dx}{dt} = \frac{\sqrt{3}}{2}x(4) \Rightarrow \frac{dA}{dt} = 2\sqrt{3}x$

when $x = 14$ cm, then $\frac{dA}{dt} = 2\sqrt{3}(14) = 28\sqrt{3}\ \mathrm{cm}^2$

Any option is not matching

Question25

Approximate change in the volume V of a cube of side x metres caused by increasing the side by 3% is

KCET 2018

Options:

A. $0.09x^3$ m^3

B. $0.03x^3$ m^3

C. $0.06x^3$ m^3

D. $0.04x^3$ m^3

Answer: A

Solution:

The explanation calculates the approximate change in the volume of a cube when its side length is increased by 3%.

Volume of a Cube: The volume V of a cube with side length x is given by the formula:

$V = x^3$

Derivative of the Volume: The rate of change of the volume with respect to the side length x is:

$\frac{dV}{dx} = 3x^2$

Change in Side Length: Since the side length is increased by 3%, the change in x is:

$dx = 3\%$ of $x = 0.03x$

Approximate Change in Volume: The approximate change in the volume, dV, is calculated using the derivative:

$dV = \left(\frac{dV}{dx}\right) \cdot dx = 3x^2 \cdot 0.03x$

This simplifies to:

$dV = 0.09x^3\,\mathrm{m}^3$

Therefore, the approximate change in the volume of the cube is $0.09x^3\,\mathrm{m}^3$.

Question26

The maximum value of $\left(\frac{1}{x}\right)^x$ is

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Options:

A. e

B. e^e

C. $e^{1/e}$

D. $\left(\frac{1}{e}\right)^{1/e}$

Answer: C

Solution:

To find the maximum value of the function $f(x) = \left(\frac{1}{x}\right)^x$, we start by rewriting it in a form that is easier to differentiate:

$f(x) = x^{-x} = e^{-x \log x}$

Next, we differentiate $f(x)$ with respect to x:

$f'(x) = e^{-x \log x}(-1 - \log x)$

Simplifying the expression, we have:

$f'(x) = -x^{-x}(1 + \log x)$

To find the critical points for potential maxima or minima, we set $f'(x) = 0$:

$-x^{-x}(1 + \log x) = 0$

This implies:

$1 + \log x = 0$

Solving for x, we get:

$\log x = -1 \quad \Rightarrow \quad x = e^{-1} = \frac{1}{e}$

The function $f(x)$ attains a maximum value at this point. Substituting back to find the maximum value:

$f\left(\frac{1}{e}\right) = \left(\frac{1}{1/e}\right)^{1/e} = e^{1/e}$

Thus, the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$.

Question27

$f(x) = x^x$ has stationary point at

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Options:

A. $x = e$

B. $x = \frac{1}{e}$

C. $x = 1$

D. $x = \sqrt{e}$

Answer: B

Solution:

To find the stationary point of the function $f(x) = x^x$, we begin by expressing it in terms of the exponential function:

$f(x) = x^x = e^{x \log x}$

Next, we calculate the derivative of $f(x)$:

$f'(x) = \frac{d}{dx}(e^{x \log x}) = e^{x \log x} \cdot (1 + \log x)$

For a stationary point, the derivative $f'(x)$ must be equal to zero:

$e^{x \log x} \cdot (1 + \log x) = 0$

Since $e^{x \log x}$ is always positive and can never be zero, this implies:

$1 + \log x = 0$

Solving for x, we find:

$\log x = -1 \quad \Rightarrow \quad x = \frac{1}{e}$

Therefore, the function $f(x) = x^x$ has a stationary point at $x = \frac{1}{e}$.

Question28

The function $f(x) = x^2 + 2x - 5$ is strictly increasing in the interval

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Options:

A. $(-\infty, -1)$

B. $[-1, \infty)$

C. $(-\infty, -1]$

D. $(-1, \infty)$

Answer: B

Solution:

To determine where the function $f(x) = x^2 + 2x - 5$ is strictly increasing, we first calculate the derivative of the function:

$f'(x) = \frac{d}{dx}(x^2 + 2x - 5) = 2x + 2 = 2(x + 1)$

A function is increasing where its derivative is greater than or equal to zero. Thus, we analyze the inequality:

$f'(x) \geq 0 \quad \Rightarrow \quad 2(x + 1) \geq 0$

Simplifying this inequality, we find:

$x + 1 \geq 0 \quad \Rightarrow \quad x \geq -1$

Therefore, the function $f(x) = x^2 + 2x - 5$ is strictly increasing on the interval $[-1, \infty)$.

Question29

The point on the curve $y^2 = x$ where the tangent makes an angle of $\pi/4$ with X-axis is

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Options:

A. $(4, 2)$

B. $\left(\frac{1}{2}, \frac{1}{4}\right)$

C. $\left(\frac{1}{4}, \frac{1}{2}\right)$

D. $(1,1)$

Answer: C

Solution:

We have, $y^2 = x \quad \ldots(i)$

$$\therefore \quad 2y\frac{dy}{dx} = 1$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2y}$$

$\therefore$ Slope of tangent $= \frac{1}{2y} \quad \ldots(ii)$

Now, tangent makes an angle of $\pi/4$ with X-axis

$\therefore$ Slope of tangent $= \tan\frac{\pi}{4} = 1 \quad \ldots(iii)$

From Eqs. (ii) and (iii), we get

$\frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2}$

Putting, $y = \frac{1}{2}$ in Eq. (i), we get

$$\left(\frac{1}{2}\right)^2 = x$$
$$\Rightarrow \quad x = \frac{1}{4}$$

$\therefore$ Required point is $\left(\frac{1}{4}, \frac{1}{2}\right)$

Question30

The rate of change of volume of a sphere with respect to its surface area when the radius is 4 cm is

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Options:

A. $4\ \mathrm{cm}^3/\mathrm{cm}^2$

B. $6\ \mathrm{cm}^3/\mathrm{cm}^2$

C. $2\ \mathrm{cm}^3/\mathrm{cm}^2$

D. $8\ \mathrm{cm}^3/\mathrm{cm}^2$

Answer: C

Solution:

Let the radius of the sphere be a

$$\therefore \text{ Volume, } V=\frac{4}{3}\pi a^3$$
$$\Rightarrow \quad \frac{dV}{da}=\frac{4}{3}\pi\left(3a^2\right)=4\pi a^2$$

Again, surface area, $s=4\pi a^2$

$$\Rightarrow \quad \frac{ds}{da}=4\pi(2a)=8\pi a$$
$$\therefore \quad \frac{dV}{ds}=\frac{(dV/da)}{(ds/da)}$$
$$=\frac{4\pi a^2}{8\pi a}=\frac{a}{2} \qquad [\because a=4\text{ cm}]$$
$$=\frac{4}{2}$$
$$=2\text{ cm}^3/\text{cm}^2$$

Question31

The value of c in mean value theorem for the function $f(x)=x^2$ in $[2,4]$ is

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Options:

A. 3

B. $7/2$

C. 4

D. 2

Answer: A

Solution:

For mean value theorem

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$\therefore \quad f(x) = x^2$

$\Rightarrow \quad f'(x) = 2x$

$\therefore \quad 2c = \frac{f(4)-f(2)}{4-2}$

$\Rightarrow \quad 2c = \frac{4^2-2^2}{4-2}$

$\Rightarrow \quad 2c = 4+2$

$\Rightarrow \quad 2c = 6$

$\Rightarrow \quad c = 3$
